Question 1.

(a) 
\[
\frac{dy}{dx} = 20x^3 + 3 + 3x^{-4}.
\]

(b) Use the chain rule:
\[
y = u^{-\frac{1}{3}}, \quad u = x + 1
\]
Thus
\[
\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = -\frac{1}{3}u^{-\frac{4}{3}} (1)
\]
\[
= -\frac{1}{3}(x + 1)^{-\frac{4}{3}}
\]

(c) Another chain rule:
\[
y = e^u, \quad u = \sqrt{x + 1}
\]
giving
\[
\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \frac{1}{2\sqrt{x + 1}} e^{\sqrt{x+1}}
\]

(d) Use the product rule:
\[
y = uv, \quad u = e^{3x}, \quad v = \ln x
\]
Thus
\[
\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx} = \ln x 3e^{3x} + e^{3x} \frac{1}{x}
\]
\[
= (3\ln x + 1/x) e^{3x}.
\]

(e) The chain rule:
\[
\frac{dy}{dx} = \frac{1}{x^2 + x^{-2}} (2x - 2x^{-3}) = \frac{2(x^2 - 1)}{x}.
\]

(f) The quotient rule:
\[
\frac{dy}{dx} = -\frac{1 - 2 \sin x}{(1 - \cos x)^2}.
\]
Question 2.

\[ \frac{dy}{dx} = -e^x \sin x + e^x \cos x \]

so

\[ \frac{d^2y}{dx^2} = -e^x \sin x - e^x \cos x + e^x \cos x - e^x \sin x = -2e^x \sin x. \]

Question 3.

From question 5(a)

\[ \frac{d^2y}{dx^2} = 60x^2 - 12x^{-5}. \]

Thus

\[ \frac{d^3y}{dx^3} = 120x + 60x^{-6}. \]

and

\[ \frac{d^4y}{dx^4} = 120 - 360x^{-7}. \]

Question 4.

The derivative is

\[ \frac{dy}{dx} = 6x^2 + 3. \]

Thus when \( x = 1, y = 4 \) and \( \frac{dy}{dx} = 9 \) and we need to find the straight line through the point \( (1, 9) \) with slope 9. The line is

\[ y = 9x - 5. \]