Question 1  

\[ P(X \leq \nu_{0.8}) = 0.8 \]

\[ P(X \leq X_{(i)}) = \frac{i}{n + 1} \]

\[ P(X \leq X_{(16)}) = \frac{16}{19 + 1} = 0.8 \]

\[ \Rightarrow \nu_{0.8} = X_{(16)} = 150 \]

Question 2  

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Range</th>
<th>25%'ile</th>
<th>50%'ile</th>
<th>75%'ile</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.17 - 0.07 = 2.10</td>
<td>0.45</td>
<td>0.77</td>
<td>1.13</td>
</tr>
<tr>
<td>2</td>
<td>4.29 - 0.74 = 3.55</td>
<td>1.37</td>
<td>2.21</td>
<td>2.88</td>
</tr>
</tbody>
</table>

Question 3  

\[ P(F) = 0.52 \]

\[ P(C) = 0.05 \]

\[ P(C \cap F) = 0.02 \]

(a)  

\[ P(F|C) = \frac{P(C \cap F)}{P(C)} = \frac{0.02}{0.05} = 0.4 \]

[b]  

(b)  

\[ P(C|F) = \frac{P(C \cap F)}{P(F)} = \frac{0.02}{0.52} = 0.038 \]
Question 4

(a) Interval is (3.8, 6.2), range is 6.2 − 3.8 = 2.4

(b) \( \bar{X} = 5. \)

(c) \( P(\mu - \sigma < X < \mu + \sigma) = 0.68 \Rightarrow \sigma = \frac{2.4}{2} = 1.2 \)

Question 5

\( P(50 < X < 80) = P(X < 80) - P(X < 50) = 0.67 - 0.18 = 0.49 \)
Question 6 [4 marks]
Rewrite the table so that it readily shows differences amongst departments with respect to acceptances.

<table>
<thead>
<tr>
<th>Dept</th>
<th>Rate</th>
<th>Tot</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.66</td>
<td>600</td>
</tr>
<tr>
<td>C</td>
<td>0.64</td>
<td>800</td>
</tr>
<tr>
<td>A</td>
<td>0.44</td>
<td>400</td>
</tr>
<tr>
<td>D</td>
<td>0.35</td>
<td>400</td>
</tr>
<tr>
<td>E</td>
<td>0.27</td>
<td>200</td>
</tr>
<tr>
<td>F</td>
<td>0.24</td>
<td>500</td>
</tr>
</tbody>
</table>

Question 7 [4 marks]

<table>
<thead>
<tr>
<th>Rank</th>
<th>Full Professor</th>
<th>Associate Professor</th>
<th>Senior Lecturer</th>
<th>Lecturer</th>
<th>(P(A_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_1) (Under 30)</td>
<td>0.002</td>
<td>0.003</td>
<td>0.049</td>
<td>0.005</td>
<td>0.059</td>
</tr>
<tr>
<td>(A_2) 30-39</td>
<td>0.045</td>
<td>0.146</td>
<td>0.140</td>
<td>0.015</td>
<td>0.346</td>
</tr>
<tr>
<td>(A_3) 40-49</td>
<td>0.134</td>
<td>0.107</td>
<td>0.052</td>
<td>0.005</td>
<td>0.298</td>
</tr>
<tr>
<td>(A_4) 50-59</td>
<td>0.125</td>
<td>0.058</td>
<td>0.031</td>
<td>0.003</td>
<td>0.217</td>
</tr>
<tr>
<td>(A_5) (Over 60)</td>
<td>0.064</td>
<td>0.013</td>
<td>0.003</td>
<td>0.000</td>
<td>0.080</td>
</tr>
<tr>
<td>(P(R_j))</td>
<td>0.370</td>
<td>0.327</td>
<td>0.275</td>
<td>0.028</td>
<td>1</td>
</tr>
</tbody>
</table>

Question 8 [3 marks]

(a) \(P(X = 10) = 0.12\) [1 mark]
(b) \(P(X \leq 10) = 0.24\) [1 mark]
(c) \(P(X > 15) = 0.05\) [1 mark]
Question 9

probability function

distribution function

(a) \( P(X = 10) = 0.08 \) [1 mark]
(b) \( P(X > 6) = 0.58 \) [1 mark]
(c) \( P(6 < X \leq 8) = 0.70 - 0.42 = 0.28 \) [1 mark]

Question 10

\[
\begin{array}{c|cccccccc}
X & 48 & 34 & 28 & 66 & 87 & 58 \\
Z & -1.2 & -2.6 & -3.2 & 0.6 & 2.7 & -0.2 \\
|Z| > 2 & T & F & F & T & F & T \\
\end{array}
\]

Question 11

\[
\begin{align*}
\text{se}(\bar{X}) &= \sqrt{\frac{15^2}{25}} = 3 \Rightarrow \bar{X} \sim N(40, 3) \\
P(37 < \bar{X} < 43) &= P\left(\frac{37 - 40}{3} < Z < \frac{43 - 40}{3}\right) \\
&= P(-1 < Z < 1) = 0.68
\end{align*}
\]

Question 12

\[X \sim N(\mu, \frac{\sigma^2}{n}) \Rightarrow \bar{X} \sim N(30, \frac{5^2}{50}) \text{ or } \bar{X} \sim N(30, \frac{1}{2})\]

Question 13

\[
\hat{p}_1 - \hat{p}_2 = \frac{18}{50} - \frac{8}{50} = 0.2
\]

95% CI(\(\hat{p}_1 - \hat{p}_2\)) = \((\hat{p}_1 - \hat{p}_2) \pm 2\text{se}(\hat{p}_1 - \hat{p}_2)\)

\[= 0.2 \pm 2 \times 0.085 \]

\[= (0.03, 0.37)\]
Question 14

\[ T = \frac{\bar{X}_2 - \bar{X}_1}{\text{se}(\bar{X}_2 - \bar{X}_1)} \]
\[ = \frac{8 - 3.5}{0.85} \]
\[ = 5.3 \]

Since \( T >> t_{16}(0.99) \), conclude that observed difference is unlikely due to chance and most likely due to treatment.

Question 15

(a) For paired t-test,

\[ T = \frac{\bar{D}}{\text{se}(\bar{D})} \]
\[ = \frac{14.3}{1.25} = 11 \]

(b) Since \( T > t_9(0.95) \) conclude that treatment is effective.

Question 16

(a) | Position | Supports | Opposes | Undecided | Total |
--- | --- | --- | --- | --- |
Male | 80 | 15 | 5 | 100 |
Female | 70 | 25 | 5 | 100 |
Total | 150 | 40 | 10 | 200 |

(b) | Supports | Position | Opposes | Undecided |
--- | --- | --- | --- |
Male | \( \frac{80 \times 100}{150 \times 100} = 75 \) | \( \frac{15 \times 100}{200 \times 100} = 20 \) | \( \frac{5 \times 100}{10 \times 100} = 5 \) |
Female | \( \frac{70 \times 100}{150 \times 100} = 75 \) | \( \frac{25 \times 100}{200 \times 100} = 20 \) | \( \frac{5 \times 100}{10 \times 100} = 5 \) |

(c) Since \( p > 0.05 \), there is no evidence against independence and therefore conclude there is no evidence to suggest that the distributions of opinion categories differ between Genders.
Question 17

<table>
<thead>
<tr>
<th>bins</th>
<th>(0-5)</th>
<th>(5-10)</th>
<th>(10-15)</th>
<th>(15-20)</th>
<th>(20-25)</th>
<th>&gt; 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>28</td>
<td>34</td>
<td>15</td>
<td>11</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>E</td>
<td>21</td>
<td>24</td>
<td>18</td>
<td>13</td>
<td>9</td>
<td>15</td>
</tr>
</tbody>
</table>

(a) 

(b) df = n - 1 = 5

(c) \( P(\chi^2_5 > 14) = 0.01 \)

(d) The probability of getting \( \chi^2_5 > 14 \) by chance suggests that the proposed model is not a good match for the observed frequencies.

Question 18

(a) There exist significant differences amongst the variety means because \( P(F > 7.95) \) by chance is only 0.006.

(b) \( \text{se}(\bar{X}) = \sqrt{\frac{S^2}{n}} = \sqrt{\frac{1.23}{5}} = 0.5 \)

(c) The CI’s are \( \bar{X}_i \pm 2.2 \times 0.5 \)

\[
17 \pm 1.1 = (15.9, 18.1) \\
19 \pm 1.1 = (17.9, 20.1) \\
20 \pm 1.1 = (18.9, 21.1)
\]

(d) 

[5 marks]

[2 marks]

[1 mark]

[1 mark]

[1 mark]

[7 marks]

[1 mark]

[1 mark]

[3 marks]

[2 marks]
Question 19

(a) Predicted stopping distance for 15 m.p.h is approximately 41 feet. [1 mark]

(b) 5% ile is -6.6. 95% ile is +6.6 [2 marks]

(c) 90% confidence interval for \((\hat{y}|x = 15)\) is 41 ± 6.6 = (34.4, 47.6) [2 marks]