Question 1

The tricky part is doing the 3D plot. Suppose the vector \( \mathbf{r} \) consists of 100002 random numbers, (10002 numbers will do nearly as well), generated by \textsc{randu} (you can take 1 as the seed). The vectors of successive values are:

\[
\begin{align*}
\text{-->x} &= \mathbf{r}(1:100000); \\
\text{-->y} &= \mathbf{r}(2:100001); \\
\text{-->z} &= \mathbf{r}(3:100002);
\end{align*}
\]

We want to plot these vectors as points in 3 dimensions. Assuming \( x, y \) and \( z \) are row vectors, the following command does the job:

\[
\text{-->param3d1}(x',y',\text{list}(z',-1))
\]

(If you can find something better, or clearer, let me know. The -1 at the end is shape of the points; you can experiment with this.) You will need to use the 3D Rot. button to rotate the graph slightly and make the planes visible.

Question 2

Your function for the inverse transform method may look like:

\[
\begin{align*}
\text{function } x &= \text{randsq1}(n) \\
&\quad \text{y = rand}(1,n) \\
&\quad x = \ldots. \\
&\end{align*}
\]

// the inverse of the distribution function

endfunction

There is an example of writing a function for an acceptance-rejection in §13.4.2 of Lecture 13, although for the assignment question you won’t need to write a function for the density.
Questions 3 and 4

There are similar examples in §14.1.3 and §14.1.4 of Lecture 14. Make sure your random numbers are in the correct range. The example in §14.1.3 is unusual in that we wanted four $[0, 1]$ random numbers.

Question 5

Use the birthdays function from Lecture 14 and experiment until you find the right $N$.

Question 6

The first thing to do is to simplify the geometry of the problem:

1. We may assume that the centre of the needle lies between two lines. The position of the needle is described by the distance from the top line, say $x$ where $0 < x < 1$, and the angle the needle makes with the vertical, say $\theta$ where $0 \leq \theta < 2\pi$.

2. Using the symmetry between the top and bottom lines, we may assume that the needle satisfies $0 < x < 1/2$. Using rotational symmetry we may assume $0 \leq \theta < \pi/2$.

The next thing to do is to determine, from elementary geometry, the condition on $x$ and $\theta$ for the needle to intersect the (upper) line.

The perform the simulation repeat the following: (1) Generate random $x$ and $\theta$, (2) Test whether the needle intersects the line, keeping count of the number of times the needle does intersect the line.