Question 1 [2 marks]

IBM mainframes in the 1960's used a random number generator called RANDU which is a linear congruential generator with $a = 65539$, $c = 0$ and $m = 2^{31}$. Use `lcg` from Lecture 13 to generate a large number of values from RANDU, say 10000 to 100000, and plot successive triples of values as points in 3 dimensions. Produce a graph showing that the points lie on a small number of planes. How many planes are there?

Question 2 [4 marks]

Consider the probability density

$$
\rho(x) = \frac{3}{2} \sqrt{x}, \quad x \in [0, 1]
$$

Write Scilab functions to generate random numbers with this density using (a) the inverse transform method, and (b) an acceptance-rejection method. Generate 10000 random numbers using each method and present histograms of your results.

Question 3 [3 marks]

Use Monte-Carlo integration to estimate the double integral

$$
\int_\Omega e^{-\sqrt{x^2+y^2}} \, dx \, dy
$$

over the semicircular region $\Omega$ defined by

$$
x^2 + y^2 \leq 1, \quad x \geq 0.
$$
Question 4 [3 marks]

Use a Monte-Carlo method to estimate the volume of the ellipsoid

\[ x^2 + \frac{y^2}{4} + \frac{z^2}{16} \leq 1. \]

Question 5 [3 marks]

For the birthday problem from Lecture 14 estimate the number \( N \) of persons needed so that the probability of at least two persons sharing a birthday is 0.99.

Question 6 [4 marks]

**Buffon’s Needle:** A needle of length 1 unit is dropped onto a sheet of paper ruled by parallel lines 1 unit apart. What is the probability that the needle intersects one of the lines?

In the simulation assume that the position of the centre of the needle is uniformly distributed between two lines and that the angular orientation of the needle is also uniformly distributed. [Hint: you will need to generate two random numbers, say \( x \) and \( \theta \), to represent the position and orientation of the needle. You will need to work out the inequality involving \( x \) and \( \theta \) that expresses the condition that the needle intersects a line.]