AMTH142 Assignment 5

Due: 10th September

This assignment covers material in Lectures 9 to 11.
This assignment is the subject of Practical 5.

Question 1 [4 marks]
Use Scilab to plot the graphs of the following functions over the intervals indicated:

(a) \( f(x) = 2x^2 - 3x + 1, \quad x \in [0, 2] \)
(b) \( f(x) = |2x^2 - 3x + 1|, \quad x \in [0, 2] \)
(c) \( f(x) = (x - 1)(x - 4), \quad x \in [0, 4] \)
(d) \( f(x) = \frac{1}{1 + x^2}, \quad x \in [-4, 4] \)
(e) \( f(x) = x \ln x, \quad x \in [0, 1] \)
(f) \( f(x) = \frac{\sin x}{x}, \quad x \in [-4\pi, 4\pi] \)

Question 2 [2 mark]
Real numbers obey the associative law of addition:

\[ a + (b + c) = (a + b) + c \]

for all real numbers \( a, b \) and \( c \). Give an example, in Scilab, to show that this law does not hold for floating point numbers.

Hint: think \( \epsilon_{\text{mach}} \).
Question 3 [2 mark]

Which of the following operations on two positive floating point numbers can produce an overflow?

1. Addition
2. Subtraction
3. Multiplication
4. Division

In each case either give an example in Scilab demonstrating overflow or explain why overflow cannot occur.

Question 4 [7 marks]

Consider the following system of linear equations:

\[
\begin{align*}
    x_2 &= x_6 \\
x_3 &= 10 \\
    \alpha x_1 &= x_4 + \alpha x_5 \\
    \alpha x_1 + x_3 + \alpha x_5 &= 0 \\
    x_4 &= x_8 \\
    x_7 &= 0 \\
    \alpha x_5 + x_6 &= \alpha x_9 + x_{10} \\
    \alpha x_5 + x_7 + \alpha x_9 &= 15 \\
    x_{10} &= x_{13} \\
    x_{11} &= 20 \\
    x_8 + \alpha x_9 &= \alpha x_{12} \\
    \alpha x_9 + x_{11} + \alpha x_{12} &= 0 \\
    x_{13} + \alpha x_{12} &= 0
\end{align*}
\]

where \( \alpha = \sqrt{2}/2 \).

(a) Solve these equations in Scilab and estimate the accuracy of your solution.

(b) Solve these equations in Maxima.

(c) Use you solution from Maxima to determine the accuracy of the Scilab solution. How does this compare to the estimated accuracy?
**Question 5 [3 marks]**

The following problem arises in surveying. Suppose we wish to determine the altitudes \(x_1\), \(x_2\), \(x_3\) and \(x_4\) of four points. As well as measuring each altitude with respect to some reference point, each point is measured with respect to all of the others. The resulting measurements are:

\[
\begin{array}{c|c}
\hline
x_1 & 2.95 \\
x_2 & 1.74 \\
x_3 & -1.45 \\
x_4 & 1.32 \\
x_1 - x_2 & 1.23 \\
x_1 - x_3 & 4.45 \\
x_1 - x_4 & 1.61 \\
x_2 - x_3 & 3.21 \\
x_2 - x_4 & 0.45 \\
x_3 - x_4 & -2.75 \\
\hline
\end{array}
\]

These form an overdetermined set of linear equations. Find the least-squares solution. How do the computed values compare to the direct measurements?