Question 1 [6 marks]
If \( A = \{1, 2, 3\} \), \( B = \{2, 4, 6, 8\} \) and the universal set \( U = \{1, 2, 3, \cdots \} \) find
(a) \( A \cap B \)
(b) \( B' \)
(c) \( \mathcal{P}(A) \), the power set of \( A \)
(d) \( A \times A \).

Question 2 [7 marks]
Prove by mathematical induction that for all \( n \in \mathbb{N}, n \geq 1 \),
\[
\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}.
\]

Question 3 [6 marks]
Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) be given by
\[
f(x) = 2x^3 - x^2 + 1.
\]
Prove from the definition that \( f(x) = O(x^3) \).

Question 4 [9 marks]
(a) Write down a truth table to show that \( p \rightarrow q \) is equivalent to \( \sim p \lor q \).
(b) Use De Morgan’s Laws to simplify
\[
\sim (\sim p \lor \sim q).
\]
(c) Show that the argument
\[
p \lor q \rightarrow r, \quad s \lor (\sim q), \quad t, \quad (\sim p) \rightarrow (\sim t), \quad p \land r \rightarrow (\sim s), \quad \therefore \sim q
\]
is valid by deducing the conclusion from the premises step by step through the use of the basic rules of inference or laws of logic.
**Question 5  [8 marks]**

(a) Represent mathematically the following statement.

A natural number, if divided by a natural number, may not remain a natural number.

(b) For the following statement, find its negation and simplify your answer. Is this negated statement true? Explain your answer.

\[ \forall x \in \mathbb{R}, \; (x > 2) \longrightarrow (x^2 > 3) \]

**Question 6  [5 marks]**

Use the Quick Sort algorithm to sort the following list of numbers, smallest number first. Use the first element as a pivot. Underline the pivot elements and use an asterisk to mark those elements that are in their final positions. How many comparisons are needed?

8, 6, 4, 7, 5, 9.

**Question 7  [10 marks]**

(a) Consider the following graph

(i) Find the following, if they exist.

   (1) An Eulerian path.

   (2) A Hamiltonian path.

(ii) Give the adjacency matrix for the graph

   \[ \begin{pmatrix} * & 1 & 1 & 1 \\ 1 & * & 1 & 1 \\ 1 & 1 & * & 1 \\ 1 & 1 & 1 & * \end{pmatrix} \]

   Question 7(b) is on page 4
Question 7 continued

(b) Explain why the graphs below are not isomorphic.

\[ G_1 \quad G_2 \]

Question 8  \([10\text{ marks}]\)

(a) Draw a binary tree to represent the following mathematical expression.

\[(a \ast b)/(c - d \ast b)\]

(b) Write down the vertex sequence for the post order traversal of the tree in (a).

(c) Use Kruskal’s algorithm to find a minimal spanning tree for the following weighted graph where the numbers represent the weight of the corresponding edge. What is the total weight of the minimal spanning tree? Draw the minimal spanning tree.

\[ v_1 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_6 \]

Question 9 is on page 5
Question 9  [8 marks]

(a) Convert the following number, in base 2, to base 4 then to base 16.

\[110110.01_2\]

(b) Perform the following in base 7 (the numbers are in base 7).

\[
\begin{array}{cccc}
3 & 4 & 6 & 1 \\
5 & 2 & 2 \\
\end{array}
\]

Question 10  [12 marks]

(a) Let \( A = \{1, 2, 3, 4\} \) and a relation \( R \) on \( A \) be given by

\[R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (4,4)\}.
\]

(i) Draw the digraph of \( R \).

(ii) Is \( R \) an equivalence relation? Justify your answer.

(b) Let \( B = \{1, 2, 3\} \) and \( R \) be a binary relation on \( \mathcal{P}(B) \) defined by

\[(C, D) \in R \text{ iff } C \subseteq D.
\]

That is, \( R \) is the “subset relation”, \( \subseteq \), on \( \mathcal{P}(B) \).

(i) Show that \( R \) is reflexive, antisymmetric and transitive. That is, show that \( R \) is a partial order relation.

(ii) Draw the corresponding Hasse diagram for the relation \( R \).

(iii) Give the least element and the greatest element, if they exist.

Question 11  [8 marks]

(a) Draw the logic gate implementation of \( xy' + x' \).

(b) Use a Karnaugh map to find a minimal representation for the following Boolean expression.

\[xyz + x'y'z + x'y'z' + x'y'z'. \]
Question 12  [11 marks]

(a) Find the solution of the recurrence relation

\[ a_{n+2} + 8a_{n+1} - 9a_n = 0, \quad n \geq 0 \]

satisfying the initial conditions

\[ a_0 = 0 \quad \text{and} \quad a_1 = 10. \]

(b) Find the general solution of the recurrence relation

\[ a_{n+2} - 5a_{n+1} + 6a_n = 3^n, \quad n \geq 0. \]