Question 1  [6 marks]

(a) If $X = \{1, 3, 5, 7\}$, $Y = \{2, 3, 5\}$ and $Z = \{1, 5, 7\}$ find

(i) $X \cap Y$

(ii) $X - Y$

(iii) $(X \cap Z) \cup Y$.

(b) If $W = \{a, b, c, d\}$, give an example of a partition of $W$ that has $\{a\}$ as one of its elements.

Question 2  [6 marks]

Prove by mathematical induction that for all $n \in \mathbb{N}$, $n \neq 0$,

$$2n \leq 2^n.$$ 

Question 3  [7 marks]

Let $f : \mathbb{N} \rightarrow \mathbb{R}$ be defined by

$$f(n) = \frac{n^4 + \log_2 n}{n^2 + 1}.$$ 

Prove from the definition that $f(n) = \Theta(n^2)$.

Question 4  [9 marks]

(a) Write down a truth table to show that $\sim (p \rightarrow q)$ is equivalent to $p \land (\sim q)$.

(b) Show that the argument

$u \rightarrow q, \ r \lor s, \ \sim s \rightarrow \sim p, \ \left((\sim p) \land r\right) \rightarrow u, \ \sim s, \ \vdots \ q$

is valid by deducing the conclusion from the premises step by step through the use of the basic rules of inference or laws of logic.
Question 5  [8 marks]

(a) Express the following as a sentence in English. Is the statement true? Explain your answer.

\[ \exists m \in \mathbb{N}, \; \forall n \in \mathbb{N}, \; m = m + n \]

(b) For the following statement, find its contrapositive. Simplify your answer.

\[ \forall x \in \mathbb{R}, \; (x \leq 0) \longrightarrow (x + 1 \leq 1) \]

Question 6  [5 marks]

Use the Insertion Sort algorithm to sort the following list of numbers, smallest number first.

6, 4, 2, 5, 3, 7

How many comparisons are needed? Give the intermediate list whenever a number has been completely inserted.

Question 7  [10 marks]

(a) Consider the following graph.

(i) Give the adjacency matrix for the graph.

(ii) Find the following if they exist

(1) an Eulerian circuit

(2) a Hamiltonian circuit.

(b) A complete graph, \( K_n \), has \( n \) vertices and each pair of vertices are joined by an edge. For which values of \( n \) is the graph \( K_n \) planar? Justify your answer.
Question 8  [10 marks]

(a) Draw a binary tree to represent the following mathematical expression.

\[(a/b) - (a + c \times b)\]

(b) Write down the vertex sequence for the postorder traversal of the tree in (a).

(c) Use Kruskal’s algorithm to find a minimal spanning tree for the following weighted
graph where the numbers represent the weight of the corresponding edge. What is
the total weight of the minimal spanning tree?

Question 9  [8 marks]

Perform the following arithmetic operations.

(a) \(C7 + B8\) (base 16)

(b) \(111011 \times 111\) (base 2).

Question 10 is on page 5
**Question 10** [12 marks]

(a) Let $A = \{1, 2, 4, 10, 12\}$ and $R$ be a binary relation on $A$ defined by

\[ \forall m, n \in A \ (m, n) \in R \ \text{iff} \ m | n. \]

That is $(m, n) \in R$ if and only if $m$ divides $n$.

(i) Show that $R$ is reflexive, antisymmetric and transitive. That is, show that $R$ is a partial order relation.

(ii) Draw the corresponding Hasse diagram for the relation $R$.

(iii) List the minimal and maximal elements, if they exist.

(b) Let $B = \{0, 1, 2, 3\}$ and a relation $R$ on $A$ be given by

\[ R = \{(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)\}. \]

(i) Draw the digraph for $R$.

(ii) Is $R$ an equivalence relation? Justify your answer.

**Question 11** [8 marks]

(a) Simplify the following switching circuit by writing down the corresponding Boolean expression, simplifying it then drawing the simplified circuit.

(b) Simplify the following switching circuit by writing down the corresponding Boolean expression, simplifying it then drawing the simplified circuit.

\[ \text{Question 11(b) is on page 6.} \]
Question 11 continued

(b) The table below specifies a Boolean function $f : S \times S \times S \to S$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$f(x, y, z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Give a Boolean expression corresponding to this function.

(c) Give the logic gate implementation of $x'y' + y$.

Question 12  [11 marks]

(a) Find the solution of the recurrence relation

$$a_{n+2} + a_{n+1} - 6a_n = 0, \quad n \geq 0$$

satisfying the initial conditions

$$a_0 = 0 \quad \text{and} \quad a_1 = 5.$$

(b) Find the general solution of the recurrence relation

$$a_{n+2} + 2a_{n+1} - 8a_n = 2^n, \quad n \geq 0.$$